Designing for Combined Random Loads

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A simple analytical procedure is presented for obtaining a consistent set of internal structural design loads for aerospace vehicles subjected to random loading phenomena. The problem is approached by considering the various internal load components, such as shear, moment, and torsion as random components of a generalized internal load vector. This vector is maximized and scaled up to design levels. The resulting components of the maximized and scaled general load vector provide a rational set of internal loads for strength-check purposes.

Introduction

As aerospace structures become more efficient and usually less rigid, it is necessary to check the strength of structural components for dynamic load conditions. Many dynamic loads are random in time, such as those resulting from atmospheric turbulence, severe fluctuating acoustic pressures, or turbulent boundary-layer excitation. These loadings can induce internal force, moment, or torsion components on structural elements or assemblies that are random in time and do not have identical time histories. That is, they are random and statistically correlated to a greater or lesser degree.[1,2]

It is generally agreed that limit design strength levels for new designs should be comparable to strength levels of past successful designs. Studies of existing structures with successful service experience have been made to determine allowable limit design factors which can, in turn, be applied to rms load values for new designs to provide sufficient strength for random loading.[3,4] The rms load level resulting from a unit rms value of the excitation or driving force $\bar{A}$ is multiplied by the limit design factor to obtain a limit design load level for a new structure†

$$\sigma = U_\sigma \bar{A} \tag{1}$$

where σ = limit design internal load, U_σ = limit design load scale factor, and $\bar{A}$ = rms internal load resulting for a unit rms level of the excitation.

When two or more internal load components are important in assessing the strength of a structural element, the question arises as to how they should be accounted for as a set of simultaneously acting internal loads. Obviously, if two random load time histories were identical, that is, if they were perfectly correlated, the structure would be checked for simultaneously acting design levels of both loads. If the load components are not perfectly correlated, then lesser values should logically be used. In the following discussion, the various load components on a structural element, such as shear, moment, torsion, and axial load are considered as vector components of a random time varying combined load vector in n space where n corresponds to the number of load components considered. The probability density for the end-point of the load vector in n space is a hyperellipsoid. The approach is to determine the direction of the major and minor semiaxes of this hyperellipsoid and to apply the scale factor U_σ to the combined load vector in these directions to obtain sets of limit and ultimate design loads.

Limit Incremental Combined Loads

Consider the simultaneous time histories for moment and shear shown in Fig. 1. It is assumed that both of these time histories have been scaled to their independent design levels by the factor U_σ. The ensuing discussion will outline an approach for obtaining both limit and ultimate design loads for which the structure can be strength checked.

The time histories in Fig. 1 can be plotted point for point in nondimensional vector form as indicated in Fig. 2. The nondimensional combined load vector $\bar{F}$ can be expressed as

$$\bar{F}(t) = f_1(t)i + f_2(t)j + f_3(t)k + \ldots \tag{2}$$

wherein each component has been normalized by its own rms design level. That is,

$$\begin{aligned} f_1(t) &= F_1(t)/\sigma_1 \\ f_2(t) &= F_2(t)/\sigma_2 \\ &\vdots \end{aligned} \tag{3}$$

Now consider a unit vector $\bar{P}$ fixed in direction,

$$\bar{P} = a_1 i + a_2 j + a_3 k + \ldots \tag{4}$$

where the a's are direction cosines, that is

$$\Sigma a_i^2 = 1 \tag{5}$$

and determine the projection of $\bar{F}$ on $\bar{P}$, that is, the scalar

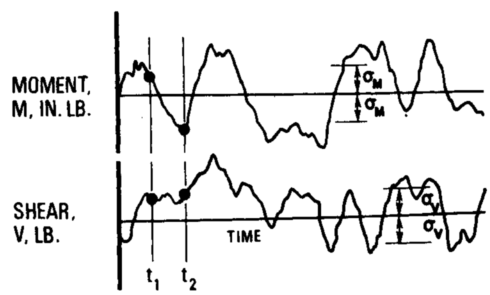

Fig. 1 Simultaneous time histories.

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† "Load component" is used here in the sense of moment, shear, torsion, or axial load to which a structural component is subjected where σ is the limit incremental design level and $\bar{A}$ is the rms value of the internal load component resulting from a unit value of the rms driving force.

product of $\bar{F}$ and $\bar{P}$

$$\bar{F}\cdot\bar{P} = a_1f_1 + a_2f_2 + a_3f_3 + \ldots \quad (6)$$

This scalar product can be squared, then averaged over time to obtain a mean-squared combined load vector in the direction of the unit vector $\bar{P}$. This procedure amounts to projecting the vector trace of $\bar{F}$ as indicated in Fig. 2, onto the position vector $\bar{P}$, then determining the mean-squared value along $\bar{P}$. Later, the direction of $\bar{P}$ is determined to maximize Eq. (6). This procedure establishes the direction for maximum incremental combined limit loads. An expression for the squared value of the scalar product is

$$(\bar{F}\cdot\bar{P})^2 = \lfloor a_1a_2a_3\cdot a_n \rfloor \begin{bmatrix} f_1^2 & f_1f_2 & f_1f_3 & . & . \\ f_1f_2 & f_2^2 & f_2f_3 & . & . \\ f_1f_3 & f_2f_3 & f_3^2 & . & . \\ . & . & . & . & . \\ . & . & . & . & f_n^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ a_n \end{bmatrix} \quad (7)$$

The mean-squared value is determined by averaging each element of the square matrix over the time domain to obtain,

$$(\bar{F}\cdot\bar{P})^2 = \lfloor a_1a_2a_3\cdot a_n \rfloor \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & . & . \\ \rho_{12} & 1 & \rho_{23} & . & . \\ \rho_{13} & \rho_{23} & 1 & . & . \\ . & . & . & . & . \\ . & . & . & . & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ a_n \end{bmatrix} \quad (8)$$

The maxima and minima for Eq. (8) can be determined by an eigenvalue solution; that is

$$\begin{bmatrix} (1-\lambda) & \rho_{12} & \rho_{13} & . & . \\ \rho_{12} & (1-\lambda) & \rho_{23} & . & . \\ \rho_{13} & \rho_{23} & (1-\lambda) & . & . \\ . & . & . & . & . \\ . & . & . & . & (1-\lambda) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ . \\ a_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

Once the eigenvalues λ are determined, each can be substituted back into Eq. (9), and the unit eigenvectors $\bar{P}$ can be determined. Consider the following example: $\bar{A}_M$ = rms moment due to unit rms excitation (200,000 lb in.), $\bar{A}_V$ = rms shear due to unit rms excitation (833 lb), U_σ = design level scale factor (60), and ρ = correlation coefficient for moment and shear (0.8).

The strength should be checked first for moment alone and shear alone; that is σ_M = (60) (200,000) = $12(10)^6$ in. lb, σ_V = (60) (833) = 50,000 lb. In addition, the structure should be checked for combined loads. The eigenvalues for the combined load vectors can be determined from the following equation corresponding to Eq. (9):

$$\begin{bmatrix} (1-\lambda) & 0.8 \\ 0.8 & (1-\lambda) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (10)$$

and the eigenvalues for the maximum and minimum eigenvectors are

$$\lambda_1 = 1.8, \quad \lambda_2 = 0.2 \quad (11)$$

If a_1 is temporarily assumed equal to unity, each eigenvalue can be substituted into Eq. (10), in turn, and the corresponding eigenvector established. Each eigenvector can be normalized to a unit vector to determine the direction cosines a_1 and a_2. That is

$$a_1^2 + a_2^2 = 1 \quad (12)$$

The first and second eigenvectors are

$$\begin{aligned} \bar{P}_1 &= \pm[1/(2)^{1/2}]i \pm [1/(2)^{1/2}]j \\ \bar{P}_2 &= \pm[1/(2)^{1/2}]i \mp [1/(2)^{1/2}]j \end{aligned} \quad (13)$$

Now, the magnitudes of the maximum and minimum mean square nondimensional load vectors can be computed

$$(\langle\bar{F}_1\cdot\bar{P}\rangle)^2 = \left\lfloor \pm\frac{1}{(2)^{1/2}} \pm \frac{1}{(2)^{1/2}} \right\rfloor \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix} \begin{bmatrix} \pm\frac{1}{(2)^{1/2}} \\ \pm\frac{1}{(2)^{1/2}} \end{bmatrix} = \pm 1.8$$

$$(\langle\bar{F}_2\cdot\bar{P}\rangle)^2 = \left\lfloor \pm\frac{1}{(2)^{1/2}} \mp \frac{1}{(2)^{1/2}} \right\rfloor \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix} \begin{bmatrix} \pm\frac{1}{(2)^{1/2}} \\ \mp\frac{1}{(2)^{1/2}} \end{bmatrix} = \pm 0.2 \quad (14)$$

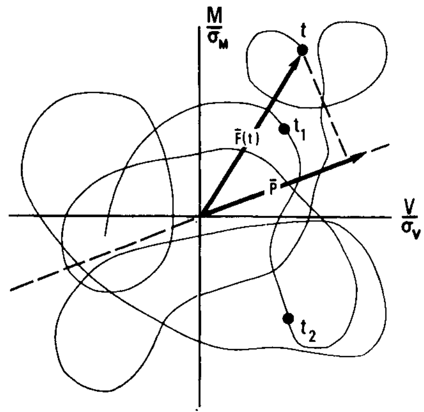

Fig. 2 Vector plot of time histories.

Obviously, the first eigenvector is in the direction of the maximum and the second is in the direction of the minimum.

The maximum nondimensional rms combined load vector has a magnitude of $(1.8)^{1/2}$ or 1.34 and the direction cosines are the same as $\bar{P}_1$, in Eq. (13). Therefore, the magnitude of the nondimensional moment is (1.34/1.414) or 0.95, and the nondimensional shear has the same value, 0.95. The combined incremental limit design loads are

$$\text{Moment} = \pm(0.95)\sigma_M = 11.4(10)^6 \text{ in. lb}$$
$$\text{Shear} = \pm(0.95)\sigma_V = 47{,}500 \text{ lb}$$

The incremental limit design load envelope for the example structure is shown in Fig. 3.

Ultimate Design Combined Loads

Ultimate design loads are loads that are not expected to be attained in service. They are determined to provide a strength capability over and above the highest expected ser-

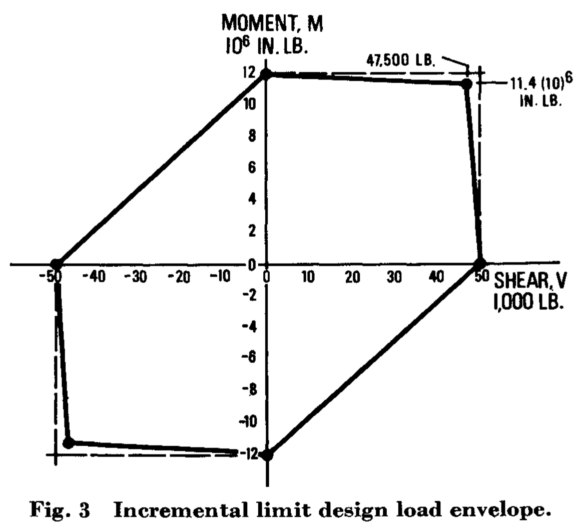

Fig. 3 Incremental limit design load envelope.

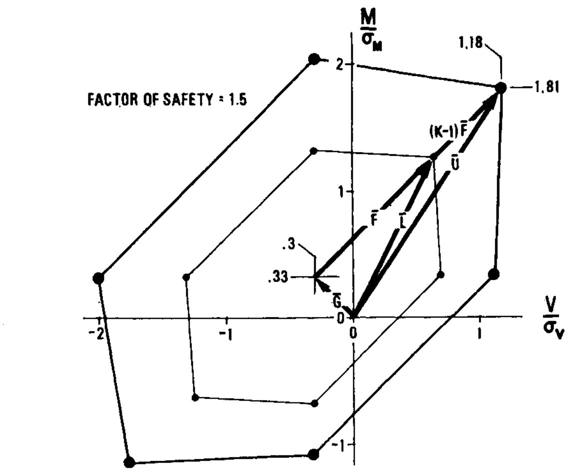

Fig. 4 Nondimensional ultimate design load envelope.

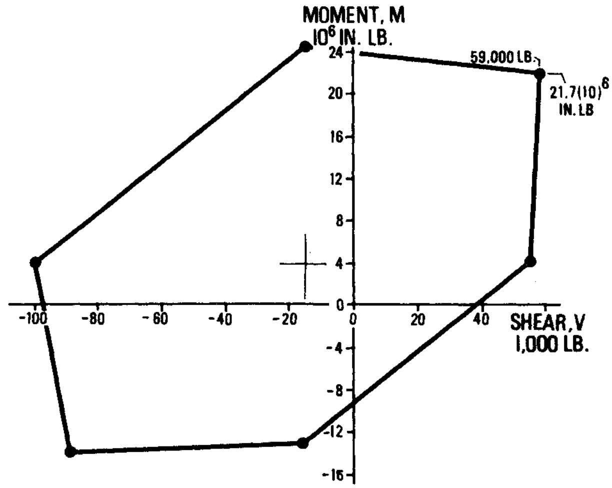

Fig. 5 Ultimate design load envelope.

vice loads, the limit loads.[5] The limit loads are total loads—incremental and steady. If the steady load vector is denoted as $\tilde{G}$, then the limit load vector can be expressed as

$$\bar{L} = \bar{G} + \bar{F} \tag{15}$$

An ultimate design combined load vector generally should not be expressed as a scalar, the factor of safety, times the limit combined load vector. The presence of nonzero steady load components should influence the direction of the ultimate load vector as the random process forcing the structure is scaled up from limit design to the ultimate design level. This argument is illustrated in Fig. 4. The steady moment was assumed to be $4(10)^6$ in. lb, and the corresponding steady shear was assumed to be $-15{,}000$ lb for the example problem. As the limit incremental design load vector $\bar{F}$ is scaled-up to ultimate, its direction does not change. The random process simply becomes more severe. Under these circumstances, the vector $\bar{F}$ should be stretched to a new length $K\bar{F}$. The extension of the vector $\bar{F}$, that is $(K - 1)\bar{F}$, should be chosen so that the ratio of the magnitudes of the ultimate vector $\bar{U}$ to the limit vector $\bar{L}$ is equal to a prescribed factor of safety S. That is,

$$|\bar{U}| = S|\bar{L}| \tag{16}$$

The value of K can be computed from the geometry indicated in Figure 4.

$$K = \frac{\{(\bar{F}\cdot\bar{F})[S^2(\bar{L}\cdot\bar{L}) - (\bar{G}\cdot\bar{G})] + (\bar{G}\cdot\bar{F})^2\}^{1/2} - (\bar{G}\cdot\bar{F})}{(\bar{F}\cdot\bar{F})} \tag{17}$$

The ultimate design load envelope accounting for the combined random loads, the steady loads and a constant factor of safety is shown in Fig. 5. It will be noted that any number of load components can be handled by this approach to the problem.

References

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